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Evidence for new seven-branes

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In order to construct globally well-defined 7-brane solutions we postulate the existence of a new type of 7-brane. We show that these new 7-branes play an important role in understanding both the existing F-theory 7-brane configurations as well as more general 7-brane configurations.

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1 Introduction

A single 7-brane forms an inconsistent background both in string theory and in supergravity. The simplest consistent supergravity 7-brane solution which has a perturbative string theory interpretation is obtained by applying two T-duality transformations to type I string theory. This background can be interpreted as the following orientifold of type IIB supergravity: $\text{Mink}_{1,7} \times T^2/\mathbb{Z}_2$. The orbifold $T^2/\mathbb{Z}_2$ has four fixed points and each corresponds to a coincident set of four D7-branes plus one O7-plane [1]. The situation in which the four D7-branes are no longer coincident is described by F-theory [2] on K3. It is known [1,3] that when the four D7-branes are separated from each other the orientifold plane splits into two non-perturbative parts, each with an $SL(2,\mathbb{Z})$ monodromy of the form $M_{1,2}TM_{1,2}^{-1}$ for some $SL(2,\mathbb{Z})$ matrix $M_{1,2}$, where $T\tau = \tau + 1$ with τ the complex axidilaton field. One of the purposes of [3] was to show that this F-theory solution can be interpreted as type IIB supergravity in the presence of a new type of 7-brane, which we refer to as the "$\det Q > 0$ 7-brane", for reasons that will become clear soon. In [3] it is shown that the F-theory 7-brane configurations form a subset of a much wider set of solutions.

The practical definition of F-theory is the statement that the axidilaton field of type IIB equals the complex structure modulus of an elliptically fibered T^2 whose base manifold is the transverse space of a collection of 7-branes. What the analysis of [3] shows is that if one accepts the existence of the $\det Q > 0$ branes and subsequently asks for the most general solution which has the property that all infinitesimal loops either have a trivial monodromy or a T monodromy then the answer is exactly the above defined F-theory solution.

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2 Seven-brane source terms

The gravity plus axidilaton system we are going to consider is a consistent truncation of the IIB supergravity action in which only the metric, the RR 0-form (axion) χ and the dilaton ϕ are kept. These two scalar fields appear in the complex combination $\tau = \chi + ie^{-\phi}$, and parameterize an $SL(2,\mathbb{R})/SO(2)$ coset.

The coupling of a 7-brane, labelled by the real numbers p, q, r, to the gravity plus axidilaton system is described by the following Einstein-frame "pseudo action" (the reason that we call the action (1) a pseudo action will become clear shortly):

$$\begin{aligned} S \quad = \quad & \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x\sqrt{-g}\left[R - \frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{2\,(\mathrm{Im}\tau)^2}\right. \\ & \left. - \int_\Sigma d^8\sigma\sqrt{-g_{(8)}}\frac{\delta(x - X(\sigma))}{\sqrt{-g}}\frac{1}{\mathrm{Im}\tau}\left(p + q|\tau|^2 + r\frac{\tau+\bar{\tau}}{2}\right)\right]\,. \end{aligned} \tag{1}$$

The 7-brane world-volume, Σ, is parameterized in the above action by $\{\sigma^i, i = 0, 1, \ldots, 7\}$. The metric on the world-volume is $g_{(8)ij}$ which is the pull-back of the target-space Einstein-frame metric $g_{\mu\nu}$. The embedding coordinates of the brane are denoted by $X^\mu(\sigma)$, and so the pull-back is given by

$$g_{(8)ij}(\sigma) = \frac{\partial X^\mu}{\partial\sigma^i}\frac{\partial X^\nu}{\partial\sigma^j}g_{\mu\nu}(X)\,. \tag{2}$$

We are only considering objects for which in the static gauge the transverse scalars are set equal to zero, i.e. we do not consider fluctuations of the world-volume. The source term in (1) should be interpreted as adding a purely static object to the theory. Note that the source term is linear in p, q and r. This is related to the fact that, unlike e.g. strings, all 7-branes have the same half-supersymmetry projection operator which is invariant under $SL(2,\mathbb{R})$ transformations.

The worldvolume term in the action is also $SL(2,\mathbb{R})$-invariant provided that the real constants p, q, r, arranged in the traceless matrix

$$Q \equiv \begin{pmatrix} r/2 & p \\ -q & -r/2 \end{pmatrix}\,, \tag{3}$$

transform in the adjoint representation of $SL(2,\mathbb{R})$. Note that the determinant of this matrix,

$$\det Q = qp - r^2/4\,, \tag{4}$$

is $SL(2,\mathbb{R})$-invariant and can be used as a label to distinguish between different $SL(2,\mathbb{R})$ conjugacy classes.

The reader may notice that the source term present in the pseudo action (1) contains only a Nambu–Goto (NG) term and no Wess–Zumino (WZ) term[1]. At first sight this seems surprising. For instance, in the case of the D7-brane, which corresponds to the case that $p = 1$ and $q = r = 0$ the source term contains only the dilaton and there is no source term for the axion whereas the D7-brane is known to have a magnetic axionic charge. The reason that we nevertheless will be able to reproduce the D7-brane solution is that we will only consider solutions for which the axidilaton τ is a holomorphic function of the two coordinates transverse to the D7-brane. This input comes from a consideration of the Killing spinor equations. Since the dilaton and axion are combined in one holomorphic function it is enough to consider a source term for the dilaton only. The action (1) is only a convenient tool for investigating supersymmetric 7-brane solutions. That is the reason that we call it a pseudo action. For the derivation of a proper action and a justification for the use of action (1) we refer to [7].

[1] For reasons explained in [7] the coupling of $\det Q > 0$ branes to type IIB supergravity is not straightforward in the $(\tau, \bar{\tau})$ parametrization of the coset manifold $SL(2,\mathbb{R})/SO(2)$; it requires a different parameterization. In [3] a trick is used to circumvent this difficulty.

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3 Seven-branes and supersymmetry

The Einstein metric and Killing spinor ϵ for the most general 7-brane solution are given by [4–6]

$$ds^2 = -dt^2 + d\vec{x}_7^{\,2} + (\mathrm{Im}\tau)|f|^2 dz d\bar{z}\,, \quad \epsilon = \left(f/\bar{f}\right)^{1/4}\epsilon_0\,, \tag{5}$$

where $z = x^8 + ix^9$ with x^8, x^9 the coordinates transverse to the 7-branes and where ϵ_0 is a constant spinor which satisfies $\gamma_{z^*}\epsilon_0 = 0$. The functions τ and f are holomorphic functions of z and are defined on the Riemann sphere. They transform under $SL(2,\mathbb{Z})$ as follows

$$\tau \to \Lambda\tau \equiv \frac{a\tau + b}{c\tau + d}\,, \quad f \to (c\tau + d)f\,, \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})\,. \tag{6}$$

The local solutions to the sourced equations of motion are characterized by the monodromy $\tau \to e^Q\tau$ where Q is the charge matrix defined in (3). The D7-brane is an element of the set of charges p, q, r which satisfy $\det Q = 0$. We assume that 7-branes for which the monodromy e^Q has trace less than 2, i. e. $\det Q > 0$, also exist.

In order to construct finite energy solutions we need to divide out type IIB supergravity by $SL(2,\mathbb{Z})$. The moduli space of this theory is given by the orbifold $\{\tau \text{ upper half-plane}\}/PSL(2,\mathbb{Z})$. Within this moduli space there are three special points (orbifold points) which are fixed points of e^Q; these are $i\infty$, $\rho = (-1 + i\sqrt{3})/2$, and i. With each fixed point of the monodromy e^Q we associate a 7-brane. The D7-brane is associated to $\tau = i\infty$, and with $\tau = \rho$, i we associate branes with some positive value of $\det Q$. Any 7-brane configuration can be considered as a certain mapping of these three orbifold points to the transverse space. This mapping is always of the following form

$$j(\tau) = \frac{P(z)}{Q(z)}\,, \tag{7}$$

where P and Q are polynomials and j is the modular j-function. The function f must by supersymmetry requirements be of the following form

$$f(z) = \eta^2(\tau)h(z)\,, \tag{8}$$

where η is the Dedekind eta function and h is a function of z which we must choose such that when going around a 7-brane f has the right monodromy. The monodromy in terms of the 7-brane charges is $\tau \to \Lambda\tau$ and $f \to (c\tau + d)f$ where $\Lambda = e^Q$, see equation (6).

The fact that the $\det Q > 0$ branes must exist can be understood by considering a solution with only one $\det Q = 0$ brane. It has been claimed in the literature that this solution is given by

$$j(\tau) = \frac{1}{z}\,, \quad f = \eta^2 z^{-1/12}\,, \tag{9}$$

with the $\det Q = 0$ brane located at $z = 0$. However, we started by stating that solutions with a single 7-brane are inconsistent, whereas this solution appears to be fine. The resolution is that the function f as given in (9) does not satisfy the right monodromy properties. In [3] it is shown that there are two inequivalent solutions with a single $\det Q = 0$ brane. The two possibilities are

$$j(\tau) = \frac{(z_i - z_{i\infty})}{(z - z_{i\infty})}\,, \quad f = \eta^2(z - z_{i\infty})^{-1/12}(z - z_i)^{-1/4}\,, \tag{10}$$

and

$$j(\tau) = \frac{(z - z_\rho)}{(z - z_{i\infty})}\,, \quad f = \eta^2(z - z_{i\infty})^{-1/12}(z - z_\rho)^{-1/6}\,. \tag{11}$$

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The first solution, equation (10), is asymptotically conical with a deficit angle of $2\pi/3$ while the asymptotic value of τ is equal to ρ. The solution has a $\det Q > 0$ brane with $-S$ monodromy at the point z_i. For the second solution, equation (11), the asymptotic value of τ is equal to i and we find asymptotically a cone with deficit angle $\pi/2$. This time there is a $\det Q > 0$ brane of $-T^{-1}S$ monodromy at the point z_ρ. With the S-duality transformation we associate the matrix

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} . \tag{12}$$

4 F-theory 7-brane configurations

The 7-brane configurations of F-theory have the property that the monodromy of τ close to the points z_i, z_ρ (defined by $\tau(z_i, z_\rho) = i, \rho$) is the identity in $PSL(2,\mathbb{Z})$ and T around $z_{i\infty}$. Further it is required that the function f has no zeros. To construct such solutions one must take coincident $\det Q > 0$ branes of opposite masses. In this case the $\det Q > 0$ branes are not noticeable from any local analysis, but they do have a non-trivial effect on the global positioning of the branch cuts, see Fig. 1 for an example of an F-theory solution with six non-trivial T-monodromies. The splitting of the O7-plane, mentioned in the introduction, can be understood from the global properties of the branch cuts ending on the $\det Q > 0$ branes.

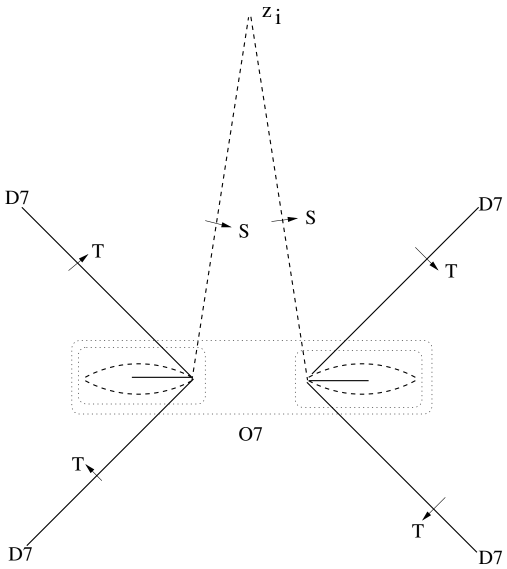

Fig. 1 F-theory solution with six non-trivial T-monodromies. The filled (dashed) lines are T (S) branch cuts.

It was mentioned in the introduction that the orientifold plane will be split into two non-perturbative parts when the $\det Q = 0$ 7-branes are separated from each other. This splitting is clear from the picture and one can read off that the monodromy around the two non-perturbative parts is $T^{-2}S^{-1}$ and $S^{-1}T^{-2}$. These monodromies can be written as $M_{1,2}TM_{1,2}^{-1}$ where the matrices $M_{1,2}$ are given by

$$M_1 = \pm \begin{pmatrix} 1 & \lambda_1 \\ -1 & 1-\lambda_1 \end{pmatrix} , \quad M_2 = \pm \begin{pmatrix} 1 & \lambda_2 \\ 1 & 1+\lambda_2 \end{pmatrix} . \tag{13}$$

What is further apparent from Fig. 1 is that the two parts of the split orientifold are each made out of one D7-brane which when approached from infinity can only be reached by crossing a branch cut.

The orientifold limit corresponds to taking the six points with T monodromy coincident. This is, however, a singular limit. There is no limit, continuous in the coupling constant, which takes one from an F-theory solution to an orientifold solution. There is an important distiction between the orientifold solutions of

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type IIB and the F-theory solutions in that in the latter case there are no 2-form potentials. An F-theory solution can be considered a solution not of type IIB but of IIB divided out by $SL(2,\mathbb{Z})$. This is because the complex axidilaton field, being a physical field, has to be single-valued and if it has the monodromy group $PSL(2,\mathbb{Z})$, all these $PSL(2,\mathbb{Z})$ transformed values must be considered equivalent.

Dividing out by $SL(2,\mathbb{Z})$ means that we are dealing with a 64+64 $N = 1, D = 8$ supergravity multiplet coupled to a 8+8 vector multiplet instead of the 128+128 IIB supergravity multiplet. There may be additional vector multiplets coming from the presence of the 7-branes in the solution. The reduction is over the two directions transverse to the 7-branes and is triggered by the fact that the transverse space dependence of the fields τ and f cannot be deformed. It may be that there are certain global obstructions which further reduce the number of fields when performing the reduction explicitly. In any case the resulting theory will be some truncation of an $N = 1, D = 8$ supergravity theory coupled to a number of vector multiplets.

The fact that in F-theory solutions or, more generally, in the solutions of [3] there cannot be any globally defined nonzero RR or NSNS 2-forms means that (p', q') strings which stretch between non-coincident $\det Q = 0$ branes are chargeless. These massive stretched strings can satisfy a BPS bound [8,9]. It would be interesting to understand the role of the $\det Q > 0$ 7-branes in the presence of these chargeless and massive (p', q') strings.

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